Entanglement, information, bulk reconstruction

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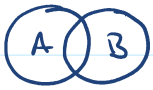
We can now compute holographically the <u>mutual information</u> for a bipartite system $\mathcal{H}_A \otimes \mathcal{H}_B$:

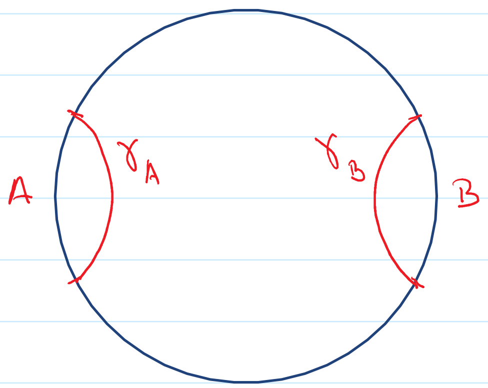
$$I(A:B) = S_A + S_B - S_{AB}$$

This is positive because of subadditivity of entanglement entropy, $S_A + S_B \geq S_{AB}$



[which in QFT, for $A \cap B \neq 0$ is satisfied because the UV divergences on the lhs are at least as big as the ones in the rhs]

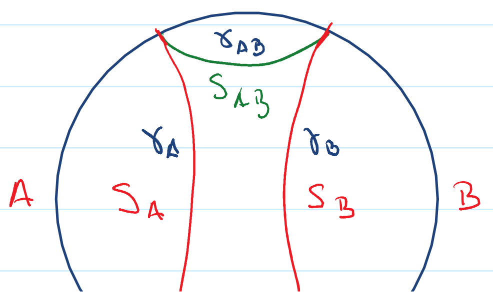
Consider the mutual info between two regions A, B in a holographic set up



$S_{AB} = S_A + S_B$

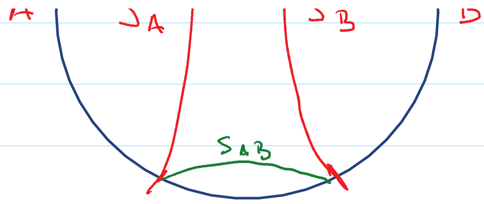
$\Rightarrow I(A:B) = 0$

but when A and B become large enough, the RT surface for $A \cup B$ changes:



$$I(A:B) = S_A + S_B - S_{AB} \neq 0$$

This means there are correlations between A and B, and the



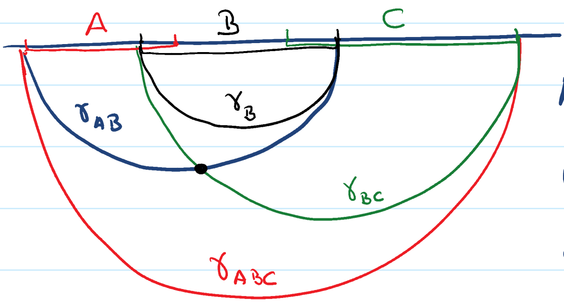
$$\text{Area}(\gamma_A) + \text{Area}(\gamma_B) > \text{Area}(\gamma_{AB})$$

between A and B, and the combined system contains more information than A and B separately
i.e. more information about how to reconstruct the bulk

Other properties of entanglement entropy become nicely geometrized by the HEE

Eg <u>strong subadditivity</u> for a tripartite system $A \cup B \cup C$

$$S_{AB} + S_{BC} - S_B \geq S_{ABC}$$



At the point where γ_{AB} and γ_{BC} intersect, split and rejoin and then deform to γ_{ABC} and γ_B.

Then, since γ_{ABC} and γ_B are minimal, it follows that

$$\text{Area}(\gamma_{AB}) + \text{Area}(\gamma_{BC}) \geq \text{Area}(\gamma_{ABC}) + \text{Area}(\gamma_B)$$

- <u>Entanglement wedge</u>

QFT state in a region A is described by a density matrix ρ_A.

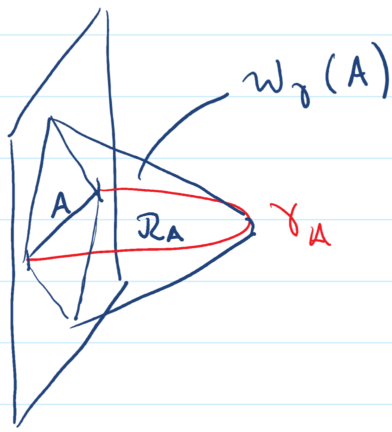
Q: What is the region of the bulk that can be reconstructed from knowledge of ρ_A?

A: The entanglement wedge $W_\gamma(A)$

Given ρ_A, it is possible to compute $S(A)$. Holographically, this is given by the area of the extremal surface γ_A.

The homology region $\mathcal{R}_A$ is enclosed between γ_A and A. The entanglement wedge $W_\gamma(A)$ is the bulk domain of dependence of $\mathcal{R}_A$:

$$W_\gamma(A) = D(\mathcal{R}_A)$$

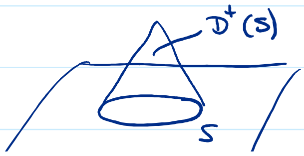


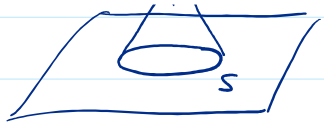
Reminder: in GR, for an achronal set S (ie no two points in S are timelike-connected) we define:

The <u>future domain of dependence</u> of S:

$$D^+(S) = \{ p \in M \mid \text{every past inextendible causal curve through } p \text{ intersects } S \}$$

inextendible ↓ not ending; causal ↓ timelike or null



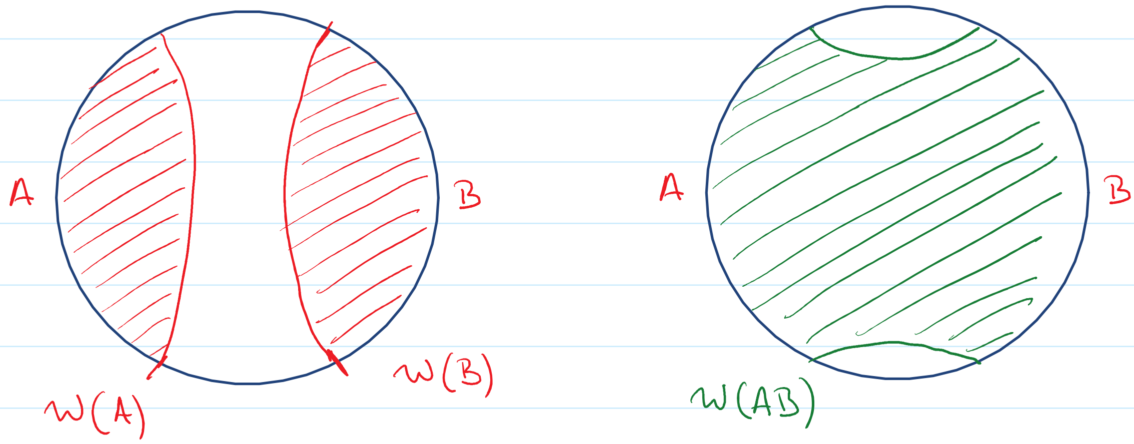


Similarly define past domain of dependence $D^-(S)$

and then domain of dependence $D(S) = D^+(S) \cup D^-(S)$

"set of events that can be determined from data on S"

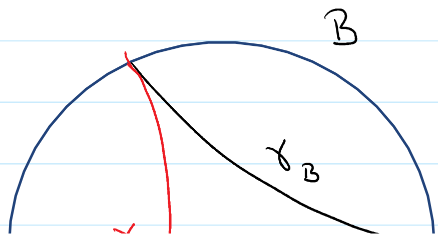
We can see that when $I(A:B) > 0$, one can reconstruct a larger region of the bulk with $A \cup B$ than with A and B separately:



Holographic quantum error correction

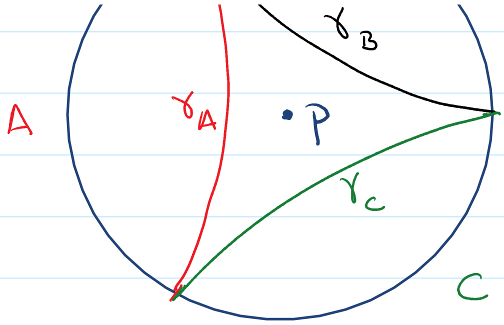
$\partial AdS = A \cup B \cup C$ A, B, C non overlapping

$\exists$ point p in the bulk cannot be reconstructed from either of A, B, C alone, but it can be from the union of any pair of them:



$p \notin W(A), W(B), W(C)$

$p \in W(AB), W(AC), W(BC)$



$p \in \mathcal{W}(AB), \mathcal{W}(AC), \mathcal{W}(BC)$

We could erase either of A, B, C and still reconstruct p

"Spacetime is a holographic quantum error-correcting code"