Basics of entanglement

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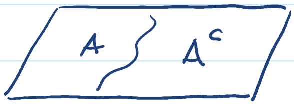
This is an abridged version of notes in "Prerequisites".

Divide a total system into disjoint subsystems A and A^c.
↳ complement of A

$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{A^c}$ Basis $|i\rangle_A |j\rangle_{A^c}$

The division can be made in many ways. One that will be of particular interest to us is a geometrical partition



[In discrete spin systems this is straightforward. Gauge theories are subtler. Continuum QFT involves Type-III vonNeumann algebras]

Operator O_A that acts only on A and is blind to A^c:

$$O_A(|i\rangle_A |j\rangle_{A^c}) = (O|i\rangle_A)\, |j\rangle_{A^c}$$

Consider now a state $|\psi\rangle \in \mathcal{H}$, and measure such operator in it:

$$\langle\psi| O_A |\psi\rangle = \mathrm{Tr}_A(\rho_A O_A)$$

with density matrix $\rho_A = \mathrm{Tr}_{A^c} |\psi\rangle\langle\psi|$

ρ_A is mixed if $\exists$ correlations between A and A^c

$$\boxed{\text{Entanglement entropy} \quad S_A = -\mathrm{Tr}_A(\rho_A \log \rho_A)}$$

measures the amount of <u>correlations between A and A^c in state $|\psi\rangle$</u>

It's a property of $|\psi\rangle$ and of the partition into A and A^c.

$\exists$ other measures of entanglement and correlations, but this is simple and useful.

When the total state is pure (as in the case above) one can easily prove that

$$S_A = S_{A^c}$$

but if the total state is mixed then they're different.

For a product state $|\psi\rangle = |\phi\rangle_A |\chi\rangle_{A^c}$

$$S_A = S_{A^c} = 0 \quad : \text{ no entanglement}$$

Examples:

- Bell states in a two-qubit system are maximally entangled

$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A |0\rangle_B \pm |1\rangle_A |1\rangle_B\right)$$

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A |1\rangle_B \pm |1\rangle_A |0\rangle_B\right)$$

$$S_A = S_B = \log 2 \quad \text{for all Bell states}$$

- In a system made of two copies of the same system

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\tilde{A}} \qquad \mathcal{H}_B \simeq \mathcal{H}_{\tilde{A}}$$

The <u>thermofield double state</u> is

$$|TFD\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n\rangle_A |\tilde{n}\rangle_{\tilde{A}} \qquad H|n\rangle = E_n |n\rangle$$

$$p_n = \frac{e^{-\beta E_n}}{Z}$$

$$S_A = \text{thermal entropy at temperature } T = 1/\beta$$