Holographic entanglement entropy: RT formula

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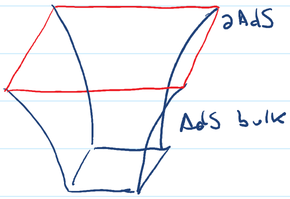
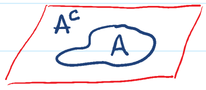
Consider a CFT with a holographic AdS dual



Partition $\Sigma = \partial AdS$
into two regions
$\Sigma = A \cup A^c$



There must be a way to compute the entanglement entropy of A when A^c is traced out by means of the bulk dual.
But it's not a priori obvious that S_{ent} should correspond to any simple geometric bulk construction.
Remarkably, it does.

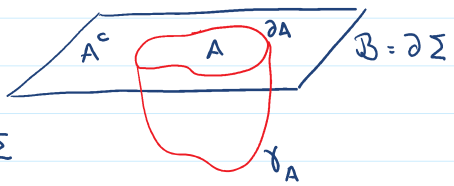
Ryu-Takayanagi:

We consider a holographic CFT_d dual to an $AAdS_{d+1}$ spacetime $\mathcal{M}$ w/ bdy $\partial\mathcal{M}$

For simplicity we take the spacetime to be static.
We take a constant time slice in the bulk, Σ with boundary $\partial\Sigma = \mathcal{B}$.

We partition $\mathcal{B} = A \cup A^c$, with entangling surface ∂A.

To compute the entanglement entropy of A, we look for a minimal bulk surface $\gamma_A \subset \Sigma$ anchored on ∂A and which is homologous to A:



- anchored on ∂A : $\gamma_A \big|_{\partial\Sigma} = \partial A$

- minimal : in Σ, γ_A has minimal area

$K = 0$ n: outward

- <u>minimal</u> : in Σ, γ_A has minimal area

  ie $\partial_n \sqrt{h}\Big|_{\gamma_A} = 0$ ie $K=0$ n: outward normal

  $\partial_n^2 \sqrt{h} > 0$

- <u>homologous to A</u> : γ_A can be continuously retracted to A

  $\exists \mathcal{R}_A \subset \Sigma$ such that $\partial \mathcal{R}_A = A \cup \gamma_A$

  ↳ smooth region

Then

$$S_A = \min_{\gamma_A} \frac{\text{Area}(\gamma_A)}{4G\hbar}$$

This is a new and deeper avatar of the Bekenstein-Hawking formula:

- <u>γ need not be a horizon</u> (although when it is, it's automatically minimal; then ∂B would be a horizon at bdry)
- The formula can be <u>proven from the gravitational path integral</u> using a replica method (Lewkowycz + Maldacena) (need not assume AdS/CFT. Generalizes Gibbons + Hawking derivation)
- This is a measure of a <u>fine-grained</u> von Neumann entropy not of a coarse-grained stat-mech entropy
- $\exists$ a <u>covariant generalization</u> to include non-static configurations: HRT (Hubeny + Rangamani + Takayanagi)
  - Consider a spatial region A in the boundary $\partial \mathcal{M}$ (This will be a codimension-2 region in $\mathcal{M}$, with a timelike and a spacelike normal)
  - Find an <u>extremal</u> bulk surface γ_A anchored on ∂A homologous to A

    The extrinsic curvatures in the two null normal directions vanish $K_{(+)} = K_{(-)} = 0$
  - Choose the one with minimal area
  - Then

$$S_A = \min_{\gamma_A}\left(\text{ext}_{\gamma_A} \frac{\text{Area}(\gamma_A)}{4G\hbar}\right)$$

An equivalent maximin prescription (Wall)

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- Consider a spatial region A in the boundary
- Choose a bulk Cauchy surface Σ_t that contains A:

  $\partial \Sigma_t = A \cup A^c$
- On Σ_t, find a <u>minimal surface</u> $\gamma_{A,t}$ of the same kind as before (anchored on ∂A, homologous to A)
- Vary the Cauchy slices Σ_t, finding minimal $\gamma_{A,t}$
- Take the one with <u>maximal area</u>

$$S_A = \max_t \min \frac{\text{Area}(\gamma_{A,t})}{4G\hbar}$$

"minimize in space

maximize in time"

- $\exists$ a <u>generalization to include quantum bulk effects</u>:

  <u>Quantum extremal surface</u> (Engelhardt + Wall)

  Accounts for quantum bulk entanglement across γ

  Given A, consider γ_A in the bulk.

  Bulk quantum fields will have an entanglement entropy S_{ent} across γ_A.

  Then extremize the "generalized entropy"

$$S_A = \min_{\gamma_A} \left\{ \text{ext}_{\gamma_A} \left(\frac{\text{Area}(\gamma_A)}{4G\hbar} + \underbrace{S_{ent}(\gamma_A)}_{\text{bulk entanglement}} \right) \right\}$$

γ_A: "quantum extremal surface"

but it is a classical geometrical surface in the bulk spacetime.

Quantum because it includes effects of quantum matter